Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 8: Integration

Contents

Questions

Question T8_Q1

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to k.

(3)

Question T8_Q2

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

Question T8_Q3

13. Show that

$$\int_0^2 2x\sqrt{x+2}\,dx = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

Question T8_Q4

10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

Question T8_Q5

13.

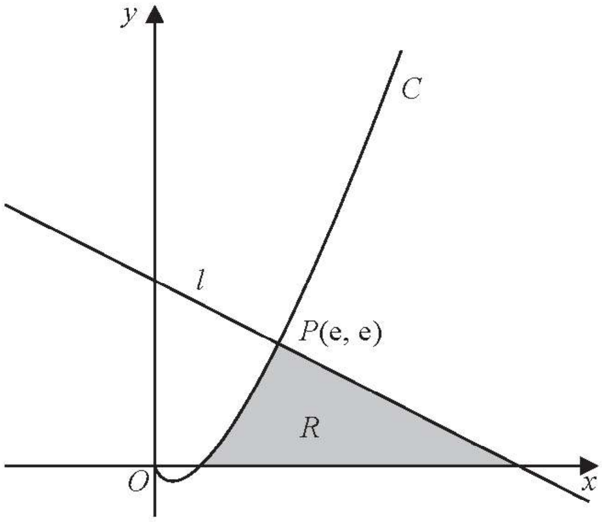


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

Question T8_Q6

8.

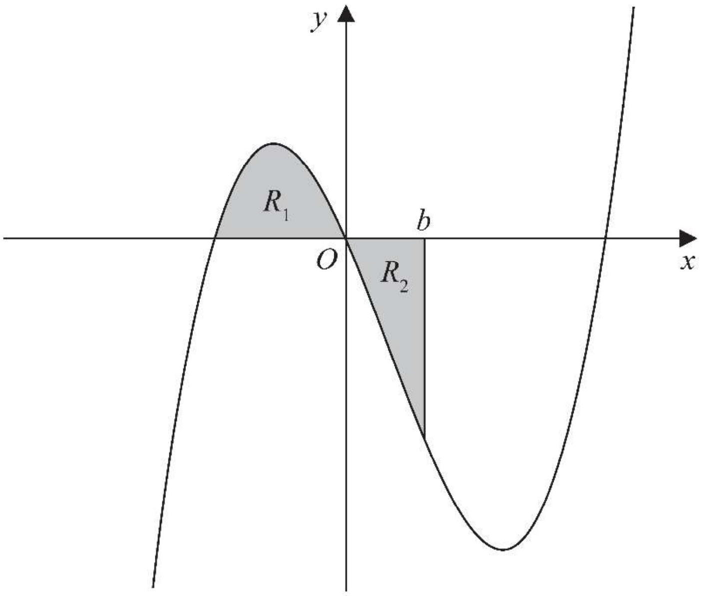


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

Question T8_Q7

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

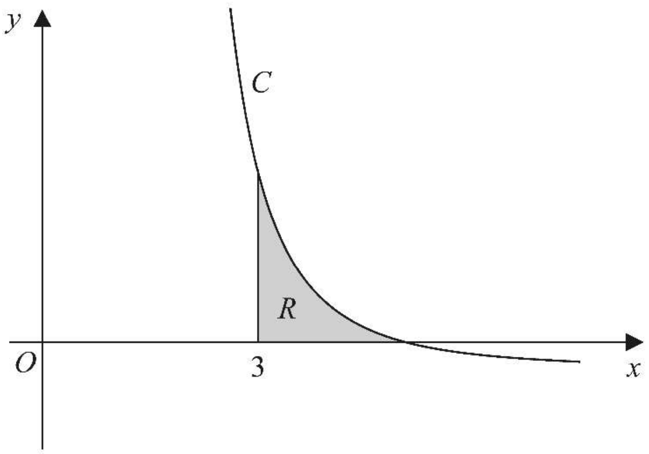


Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

Question T8_Q8

5.

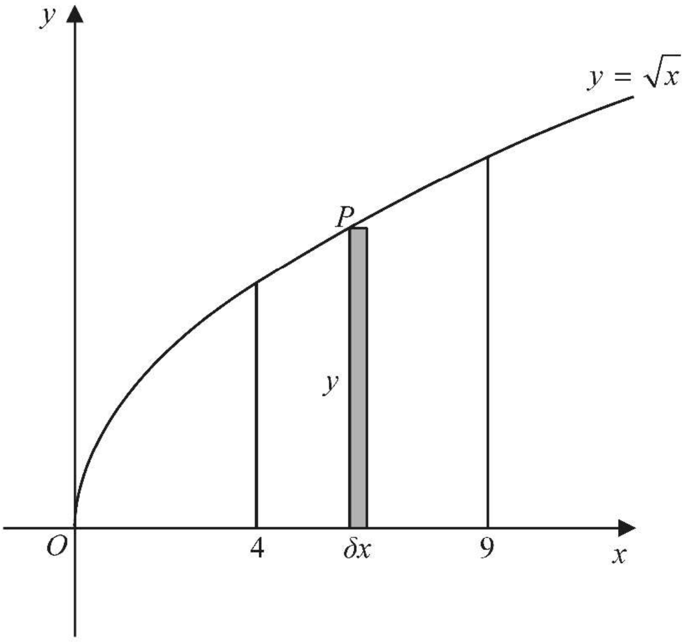


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

(3)

Question T8_Q9

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

Question T8_Q10

10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

Question T8_Q11

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

Question T8_Q12

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} \, dx$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

Question T8_Q13

8. A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(6)

Question T8_Q14

12.

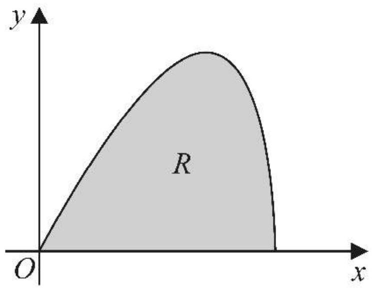


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t \, \mathrm{d}t$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

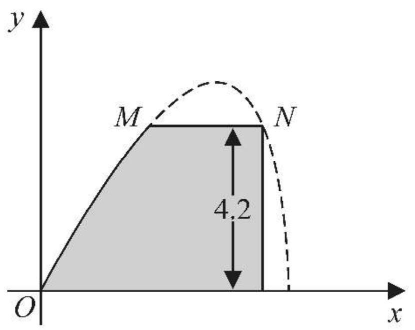


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

Question T8_Q15

12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \int_p^q \frac{2(u-1)^3}{u}\,\mathrm{d}u$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = A - B\ln 5$$

where A and B are constants to be found.

(4)

Question T8_Q16

14.

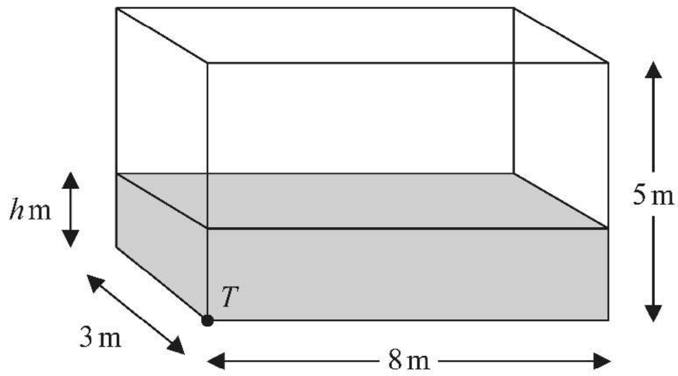


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of 0.48 m^3 per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h$ m^3 per minute

(a) Show that, according to the model,

$$1200\frac{dh}{dt} = 24 - 5h$$

(4)

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where A, B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)

Mark Scheme

Question T8_Q1

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right) \times \frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to *k*.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Question T8_Q2

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}dH = \int \frac{\cos 0.25t}{40}dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5 \Rightarrow c = \ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int \frac{1}{H}dH = \int \frac{\cos 0.25t}{40}dt$ or equivalent.

The integral signs need to be present on both sides and the dH AND dt need to be in the correct positions.

M1: Integrates both sides to reach $\ln H = A\sin 0.25t$ or equivalent with or without the $+c$

A1: $\ln H = \frac{1}{10}\sin 0.25t + c$ or equivalent with or without the $+c$. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H = 4\sin 0.25t + c$ or equivalent.

dM1: Substitutes $t=0, H=5 \Rightarrow c = ..$ There needs to have been a single "$+c$" to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H = 4\sin 0.25t + c \Rightarrow\Rightarrow H^{40} = e^{4\sin 0.25t} + e^c \Rightarrow 5^{40} = 1 + e^c \Rightarrow c = ...$

Also many students will be attempting to get to the given answer so condone the method of finding c =...
These students will lose the A1* mark

A1*: Proceeds via $\ln H = \frac{1}{10}\sin 0.25t + \ln 5$ or equivalent to the given answer $H = 5e^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone c's going to c's when they should be e^c or A

Accept as a minimum $\ln H = \frac{1}{10}\sin 0.25t + \ln 5 \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + \ln 5}$ or $H = e^{\frac{1}{10}\sin 0.25t} \times e^{+\ln 5}$ before sight of the given answer

If the only error was to omit the integration signs on line 1, thus losing the first M1, allow the candidate to have access to this mark following a correct intermediate line (see above).

If they attempt to change the subject first then the constant of integration must have been adapted if the A1* is to be awarded. $\ln H = \frac{1}{10}\sin 0.25t + c \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + c} \Rightarrow H = Ae^{\frac{1}{10}\sin 0.25t}$

The dM1 and A1* under this method are awarded at virtually the same time.

Also, for the final two marks, you may see a proof from $\int_0^H \frac{40}{H}\, dH = \int_5^t \cos 0.25t\, dt$

…….

There is an alternative via the use of an integrating factor.

…….

(b)

B1: States that the maximum height is 5.53 m Accept $5e^{0.1}$ Condone a lack of units here, but penalise if incorrect units are used.

(c)

M1: For identifying that it would reach the maximum height for the 2nd time when $0.25t = \frac{5\pi}{2}$ or 450

A1: Accept awrt 31.4 or 10π Allow if units are seen

Question T8_Q3

Question	Scheme for Substitution		Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • Using a valid substitution $u =$..., changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{dx}{du}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{dx}{du}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,dx$ $=\int A(u^2 \pm 2)u^2 du$	$\int 2x\sqrt{x+2}\,dx$ $=\int A(u \pm 2)\sqrt{u}\,du$	M1	1.1b
	$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}(2+\sqrt{2})$ *		A1*	2.1
			(7)	
				(7 marks)

M1: For attempting to integrate using substitution. Look for

- terms and limits changed to u's. Condone slips and errors/omissions on changing $dx \rightarrow du$
- attempted multiplication of terms and raising of at least one power of u by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to x's the limit is 2

B1: For substitution it is for giving the substitution and stating a correct $\frac{dx}{du}$

Eg, $u=\sqrt{x+2} \Rightarrow \frac{dx}{du}=2u$ or equivalent such as $\frac{du}{dx}=\frac{1}{2\sqrt{x+2}}$

M1: It is for attempting to get all aspects of the integral in terms of 'u'.
All terms must be attempted including the dx. You are only condoning slips on signs and coefficients

dM1: It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M

A1: Correct answer in x or u See scheme

ddM1: Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

A1*: Proceeds correctly to $=\frac{32}{15}(2+\sqrt{2})$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}(2+\sqrt{2})$

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x=Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x \to x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

A1*: Proceeds to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer.**

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

Question T8_Q4

Question	Scheme	Marks	AOs
10 (a)	$\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ (for k or a numerical k)	M1	3.3
	$\int r^2\,\mathrm{d}r = \int \pm k\,\mathrm{d}t \Rightarrow ...$ (for k or a numerical k)	M1	2.1
	$\frac{1}{3}r^3 = \pm kt \; \{+ c\}$	A1	1.1b
	$t = 0,\ r = 5$ and $t = 4,\ r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth $t = 0,\ r = 5$ and $t = 240,\ r = 3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
		(5)	
(b)	$r = 0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$	M1	3.4
	time = 5 minutes 6 seconds	A1	1.1b
		(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth	B1	3.5b
		(1)	
			(8 marks)

Notes for Question 10	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{dr}{dt} = f(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2\,dr = \int \pm k\,dt$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,dt \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t = 0,\ r = 5$ and $t = 4,\ r = 3$, or • $t = 0,\ r = 5$ and $t = 240,\ r = 3$, ***on their integrated equation*** to find their constants k and c ***and*** obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250-49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250-2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000-49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000-120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1
(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = ...$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

Question T8_Q5

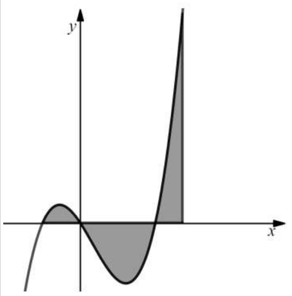
Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right)$ $\{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$)	A1	1.1b
	{Areas:} either $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ or $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = \ldots$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$, with limits of e and 1 and some attempt to substitute these and subtract • or Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$

Notes for Question 13 Continued	
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\mathrm{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\mathrm{e}\right) \mathrm{d}x = [\ \ldots\ ]_{\mathrm{e}}^{\text{their } x_A} = \ldots$
Note:	**<u>Calculator approach with no algebra, differentiation or integration seen:</u>** • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x=1$ and $x=\mathrm{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972\ldots + 7.3890\ldots = 9.4862\ldots$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Question T8_Q6

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \,\mathrm{d}x \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y\,\mathrm{d}x$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x -axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection $3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out $3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have $=0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 (x = 5.442 may not be labelled.)

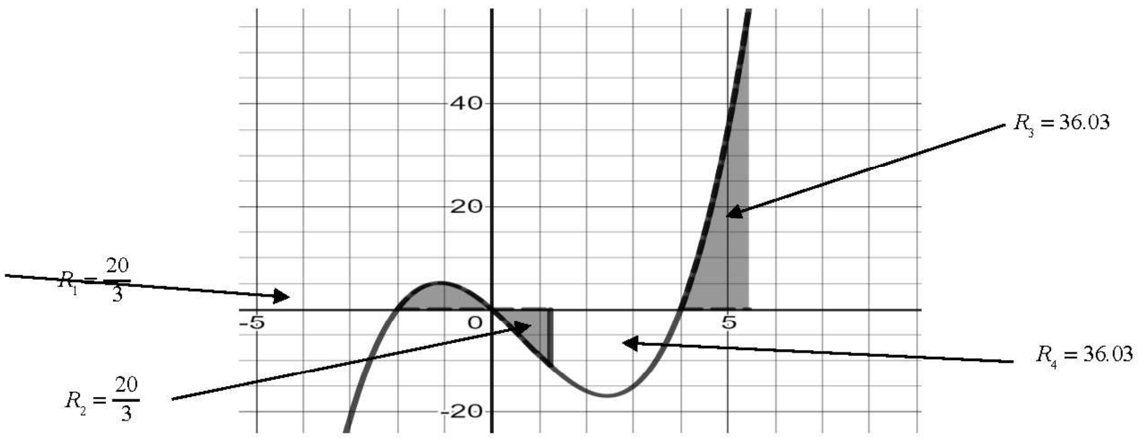
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x -axis with appropriate areas shaded or labelled.

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



Question T8_Q7

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	I = $\int\frac{15-3x}{(2x-4)(x+3)}\,dx=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	I = $\int\frac{15-3x}{(2x-4)(x+3)}\,dx=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\,dx$ Or $[.............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q=4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R=\int\frac{15-3x}{(2x-4)(x+3)}\,dx=m\ln(2x-4)+n\ln(x+3)$

OR $\int\frac{15-3x}{(2x-4)(x+3)}\,dx=m\ln(x-2)+n\ln(x+3)$

Note that $\int\frac{l}{(x-2)}\,dx \to l\ln(kx-2k)$ and $\int\frac{m}{(x+3)}\,dx \to m\ln(nx+3n)$

A1ft: $=\int\frac{15-3x}{(2x-4)(x+3)}\,dx=0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5\frac{15-3x}{(2x-4)(x+3)}(dx)$ or $[.............]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8=3\ln 2$ or $m\ln 6k-m\ln 2k=m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Question T8_Q8

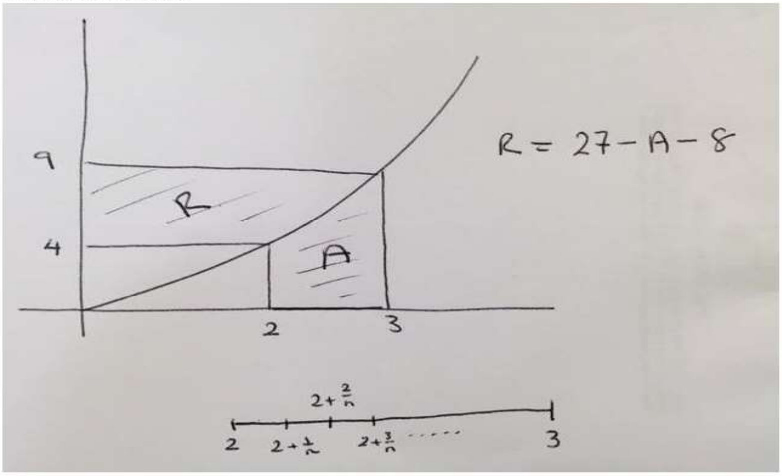
Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\ \delta x \text{ is} \right\}$ $\int_4^9 \sqrt{x}\,dx$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,dx$ with or without the 'dx'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}} \right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,dx - \int_1^3 \sqrt{x}\,dx$ or for $\int_3^9 \sqrt{x}\,dx$ without reference to a correct $\int_4^9 \sqrt{x}\,dx$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3} x^{\frac{3}{2}} + c \right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,dx$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question 5 Continued

Alt The following method is correct:



$$\text{Area}(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or} \quad \text{awrt } 12.7$$

Question T8_Q9

Question	Scheme	Marks	AOs
14 (a)	$\{u = 4 - \sqrt{h} \Rightarrow\}\ \frac{du}{dh} = -\frac{1}{2}h^{-\frac{1}{2}}$ or $\frac{dh}{du} = -2(4-u)$ or $\frac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{\int \frac{dh}{4-\sqrt{h}} = \right\}\ \int \frac{-2(4-u)}{u}du$	M1	2.1
	$= \int\left(-\frac{8}{u} + 2\right)du$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4 - \sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16,\ 0 \le h < 16,\ 0 < h \le 16,\ 0 \le h \le 16,$ $h < 16,\ h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \frac{1}{(4-\sqrt{h})}dh = \int \frac{1}{20}t^{0.25}dt$	B1	1.1b
	$-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \frac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t = 0,\ h = 1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h = 12 \Rightarrow\}\ -8\ln\left\|4-\sqrt{12}\right\| - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	
(c) Way 2	$\int_1^{12} \frac{20}{(4-\sqrt{h})}dh = \int_0^T t^{0.25}dt$	B1	1.1b
	$\left[20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h})\right]_1^{12} = \left[\frac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	
			(15 marks)

Notes for Question 14	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h,\ \mathrm{d}h = -2(4-u)\mathrm{d}u,\ \mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: Note:	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u$; $k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\};\ A, B \neq 0$
M1:	$\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \to D\ln u + Eu;\ A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int\left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \to -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$
	Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark $\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u = \int \frac{2u-8}{u}\mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u = (2u-8)\ln u - 2(u\ln u - u)\ \{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{\mathrm{d}u\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu;\ D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: Note:	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow 4-\sqrt{h} > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 14	
(c)	**Way 1**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their }1.25} = \ldots$ followed by a correct method to give $t = \ldots$; $t > 0$
Note:	$t^{\text{their }1.25} = \ldots$ can be negative, but their '$t = \ldots$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their }1.25} = \ldots$ (usually $t^{0.25} = \ldots$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{\mathrm{d}h}{\mathrm{d}t}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1: **Note:**	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

Question T8_Q10

Question	Scheme	Marks	AOs
10 (a)	$x=u^2+1 \Rightarrow \mathrm{d}x=2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int \frac{3\mathrm{d}x}{(x-1)(3+2\sqrt{x-1})}=\int \frac{3\times 2u\ \mathrm{d}u}{(u^2+1-1)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p=2, q=3$	B1	1.1b
	$=\int \frac{3\times 2\not{u}\ \mathrm{d}u}{u^{\not{2}}(3+2u)}=\int \frac{6\ \mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)}=\frac{A}{u}+\frac{B}{3+2u} \Rightarrow A=..., B=...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6\ \mathrm{d}u}{u(3+2u)}=2\ln u-2\ln(3+2u)$ $(+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u=\text{"3"}, u=\text{"2"}$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3-2\ln 9)-(2\ln 2-2\ln 7)=2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
			(10 marks)
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $\mathrm{d}x=2u\mathrm{d}u$ or exact equivalent. E.g. $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$, $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x=u^2+1$, including $\mathrm{d}x \to ...u\mathrm{d}u$ to form an integrand in terms of u. Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p=2, q=3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B.

A1: Correct PF $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$ (Not scored for just the correct values of A and B)

dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns. Look for $P\ln u+Q\ln(3+2u)$

A1ft: Correct integration for their $\frac{A}{u}+\frac{B}{3+2u} \to A\ln u+\frac{B}{2}\ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k\ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u's back to x's and use limits of 5 and 10.

A1: Proceeds to $\ln\frac{49}{36}$. Answers without working please send to review.

Question T8_Q11

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{\mathrm{d}V}{\mathrm{d}t} = -c$ (for positive constant c)	B1	3.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t} = -c$ and $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1	2.1
	$-c = 4\pi r^2 \times \frac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2} \Rightarrow \int r^2 \mathrm{d}r = \int -k \mathrm{d}t$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3} = -kt(+\alpha)$	A1	1.1b
	Uses $t = 0, r = 40 \Rightarrow \alpha = ...$ $\qquad \alpha = \frac{64000}{3}$	M1	1.1b
	Uses $t = 5, r = 20$ & $\alpha = ... \Rightarrow k = ...$	M1	3.4
	$r^3 = 64000 - 11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. $"64000 - 11200t" ... 0 \Rightarrow t...$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
		(10 marks)	
Notes:			

(a)

B1: Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant *c*).

Any "letter" is acceptable here including *k*.

Note that $\frac{dV}{dt} = c$ is B0 unless they state that *c* is a negative constant.

M1: For an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = 4\pi r^2$

Allow for an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = \lambda r^2$ (Any constant is fine)

There is no requirement to use the correct formula for the volume of a sphere for this mark.

A1*: Proceeds to the given answer with an intermediate line equivalent to $\frac{dr}{dt} = -\frac{c}{4\pi r^2}$

If candidate started with $\frac{dV}{dt} = -k$ they must provide a minimal explanation how

$\frac{dr}{dt} = -\frac{k}{4\pi r^2} \to \frac{dr}{dt} = -\frac{k}{r^2}$. E.g $\frac{1}{4\pi}$ is a constant so replace $\frac{k}{4\pi}$ with *k*

It is not necessary to use the full formula for the volume of a sphere, eg allow $V = \kappa r^3$ but if it has been quoted it must be correct. So using $V = 4r^3$ can potentially score 2 of the 3 marks.

(b)

M1: For the key step of separating the variables correctly AND integrating one side with at least one index correct. The integral signs do not need to be seen.

A1: Correct integration E.g. $\frac{r^3}{3} = -kt(+\alpha)$ or equivalent. The $+\alpha$ is not required for this mark.

This may be awarded if *k* has been given a value.

M1: Uses the initial conditions to find a value for the constant of integration α

If a constant of integration is not present, or *k* has been given a pre defined value, then only the first two marks can be awarded in part (b)

The mark may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

M1: Uses the second set of conditions with their value of α to find *k*

This may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

A1: Obtains any correct equation for the model.

E.g. $r^3 = 64000 - 11200t$ or exact equivalent such as $\frac{r^3}{3} = \frac{64000}{3} - \frac{11200}{3}t$.

ISW after sight of a correct answer. Condone recurring decimals e.g. $21333.\dot{3}$ for $\frac{64000}{3}$

Do not award if **only the** rounded/truncated decimal equivalents to say $\frac{64000}{3}$ is used.

(c)

M1: Recognises that the model is only valid when $r \geqslant 0$ **and uses this to find *t***. Condone $r > 0$

Award for an attempt to find the value of *t* when *r* =0. See scheme.

It must be from an equation of the form $ar^n = b - ct$, $a, b, c > 0$ which give + ve values of *t*.

A1ft: Allow valid for times up to (and including) $\frac{40}{7}$ seconds, 5.71 seconds. Allow $t < \frac{40}{7}$ or $t \leqslant \frac{40}{7}$

There is no requirement for the left hand side of the inequality, 0

States invalid for times greater than $\frac{40}{7}$ seconds, 5.71 seconds.

Follow through on their equation so allow $t <$ their "$\frac{64000}{11200}$" as long as this value is greater than 5

($t = 5$ is one of the values in the question)

Question T8_Q12

6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **or** $x+2\overline{)x^2+8x-3}$ with quotient $x+6$: $\underline{x^2+2x}$ $6x-3$ $\underline{6x+12}$ -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \int x+6-\frac{15}{x+2}\,\mathrm{d}x = \ldots - 15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2) \quad (+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,\mathrm{d}x = \left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ $=(18+36-15\ln 8)-(0+0-15\ln 2)$ $=18+36-(15-45)\ln 2$ or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	$=54-30\ln 2$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

M1: Multiplies by $(x + 2)$ and attempts to find values for A, B and C e.g. by comparing coefficients or substituting values for x. If the method is unclear, at least 2 terms must be correct on rhs.
Or attempts to divide x^2+8x-3 by $x + 2$ and obtains a linear quotient and a constant remainder.
This mark may be implied by 2 correct values for A, B or C

A1: Two of $A = 1$, $B = 6$, $C = -15$. But note that **just** performing the division correctly is insufficient and they must clearly identify their A, B, C to score any accuracy marks.

A1: All three of $A = 1$, $B = 6$, $C = -15$
This is implied by stating $\frac{x^2+8x-3}{x+2}=x+6-\frac{15}{x+2}$ or within the integral in (b)

(b)

M1: Integrates an expression of the form $\frac{C}{x+2}$ to obtain $k\ln(x + 2)$.
Condone the omission of brackets around the "$x + 2$"

A1ft: Correct integration ft on their $Ax+B+\frac{C}{x+2}$, $(A,B,C\neq 0)$ The brackets should be present around the "$x + 2$" unless they are implied by subsequent work.

M1: Substitutes both limits 0 and 6 into an expression that contains an x or x^2 term or both and a ln term and subtracts either way round WITH fully correct log work to combine two log terms (but allow sign errors when removing brackets) leading to an answer of the form $a + b\ln c$ (a, b and c not necessarily integers)
e.g. if they expand to get $-15\ln 8 - 15\ln 2$ followed by $-15\ln 16$ and reach $a + b\ln c$ then allow the M mark

A1: $54 - 30\ln 2$ (Apply isw once a correct answer is seen)

Question T8_Q13

8	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$f(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots\ (6)$	dM1	1.1b
	$(f(x) =)\,2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x) =)(x+4)(2x^2 - 5x - 3)$ $(f(x) =)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the $+ c$

A1: Fully correct integration (may be unsimplified). The $+ c$ must be seen (or implied by the –12)

B1: Deduces that the constant term is –12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4) = 0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set = 0.

For reference, the quotient is $2x^2 + \left(\frac{a}{2} - 8\right)x + 9 - 2a$ and the remainder is $8a - 48$

May also use $(x + 4)(px^2 + qx + r) = 2x^3 + \frac{1}{2}ax^2 - 23x - 12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(x) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4) = 0$ or long division/comparing coefficients with $(x + 4)$ as a factor.

A1cso: For $(f(x) =)\,2x^3 + 3x^2 - 23x - 12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

<u>Via firstly using factor</u>

Question	Scheme	Marks	AOs
8 Alt	$f(x) = (x+4)(Ax^2 + Bx + C)$	M1 A1	1.1b 1.1b
	$f(x) = Ax^3 + (4A + B)x^2 + (4B + C)x + 4C \Rightarrow C = -3$	B1	2.2a
	$f'(x) = 3Ax^2 + 2(4A + B)x + (4B + C)$ and $f'(x) = 6x^2 + ax - 23$ $\Rightarrow A = \ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$f(x) = (x+4)(2x^2 - 5x - 3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that f(x) is a cubic expression with a factor of $(x + 4)$

A1: For $f(x) = (x + 4)(Ax^2 + Bx + C)$

B1: Deduces that $C = -3$

dM1: Attempts to differentiate either by product rule or via multiplication and compares to $f'(x) = 6x^2 + ax - 23$ to find A.

dM1: Full method to get A, B and C

A1cso: $f(x) = (x + 4)(2x^2 - 5x - 3)$ or $f(x) = (x + 4)(2x + 1)(x - 3)$

Question T8_Q14

Question	Scheme	Marks	AOs
12(a)(i)	$y\times\frac{dx}{dt}=5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2

dM1: Attempts to find **TWO** distinct values of x when $\sin 2t=\frac{4.2}{5}$. Condone poor trig work and allow this mark if 2 values of x are attempted from 2 values of t.

A1: Both values correct to 2 dp. NB $x = 2.869\ldots,\ 5.269\ldots$

Or may take Cartesian approach

$$5\sin 2t=4.2\Rightarrow 10\sin t\cos t=4.2\Rightarrow 10\frac{x}{6}\sqrt{1-\frac{x^2}{36}}=4.2\Rightarrow x^4-36x^2+228.6144=0\Rightarrow x=2.869\ldots,5.269\ldots$$

M1: For converting to Cartesian form A1: Correct quartic M1: Solves quartic A1: Correct values

A1: 2.40 metres or 240 cm

Allow awrt 2.40 m or allow 2.4m (not awrt 2.4 m) and allow awrt 240 cm. **Units are required**.

	(Area =) $\int 5\sin 2t \times 6\cos t\,dt = \int 5\times 2\sin t\cos t \times 6\cos t\,dt$ or $\int 5\sin 2t \times 6\cos t\,dt = \int 60\sin t\cos^2 t\,dt$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,dt$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\,dt = -20\cos^3 t$	M1 A1	1.1b 1.1b
	Area $= \left[-20\cos^3 t\right]_0^{\frac{\pi}{2}} = 0-(-20) = 20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

<u>**Notes:**</u>

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A\sin 2t\cos t$ but may apply $\sin 2t = 2\sin t\cos t$ here

dM1: Attempts to use $\sin 2t = 2\sin t\cos t$ within an integral which may be implied by

e.g. $A\int \sin 2t \times \cos t\,dt = \int k\sin t\cos^2 t\,dt$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2\sin t\cos t$ or e.g. $5\sin 2t = 10\sin t\cos t$ seen <u>explicitly</u> in their proof and a correct intermediate line that includes an integral sign and the "dt"

Allow the limits to just "appear" in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60\sin t\cos^2 t\,dt = k\cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain

$\int 60\sin t\cos^2 t\,dt = ku^3$

A1: Correct integration $-20\cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0-(-20)$ and

not <u>just:</u> $-20\cos^3\frac{\pi}{2} - \left(-20\cos^3 0\right) = 20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1}\frac{4.2}{5} \Rightarrow t = ...$

A1: At least one correct value for t, correct to 2 dp. FYI $t = 0.4986...,1.072...$ or in degrees $t = 28.57..., 61.42...$

Question T8_Q15

Question	Scheme	Marks	AOs
12(a)	$u=1+\sqrt{x}\Rightarrow x=(u-1)^2\Rightarrow \frac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x}\Rightarrow \frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}2(u-1)du$ or $\int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times 2x^{\frac{1}{2}}du=\int \frac{2x^{\frac{3}{2}}}{u}du=\int \frac{2(u-1)^3}{u}du$	M1	2.1
	$\int_0^{16} \frac{x}{1+\sqrt{x}}dx=\int_1^5 \frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$2\int \frac{u^3-3u^2+3u-1}{u}du=2\int \left(u^2-3u+3-\frac{1}{u}\right)du=...$	M1	3.1a
	$=(2)\left[\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\frac{5^3}{3}-\frac{3(5)^2}{2}+3(5)-\ln 5-\left(\frac{1}{3}-\frac{3}{2}+3-\ln 1\right)\right]$	dM1	2.1
	$=\frac{104}{3}-2\ln 5$	A1	1.1b
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct expression for $\frac{dx}{du}$ or $\frac{du}{dx}$ (or u') or dx in terms of du or du in terms of dx

M1: Complete method using the given substitution.

This needs to be a correct method for their $\frac{dx}{du}$ or $\frac{du}{dx}$ leading to an integral in terms of u only (ignore any limits if present) so for each case you need to see:

$$\frac{dx}{du}=f(u)\to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}f(u)du$$

$$\frac{du}{dx}=g(x)\to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times\frac{du}{g(x)}=\int h(u)du.$$ In this case you can condone slips with coefficients e.g. allow $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}\to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times\frac{x^{\frac{1}{2}}}{2}du=\int h(u)du$

but not $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}} \rightarrow \int \frac{x}{1+\sqrt{x}}\mathrm{d}x=\int \frac{x}{u}\times\frac{x^{-\frac{1}{2}}}{2}\mathrm{d}u=\int \mathrm{h}(u)\mathrm{d}u$

A1: All correct with correct limits and no errors. The "du" must be present but may have been omitted along the way but it must have been seen at least once before the final answer. The limits can be seen as part of the integral or stated separately.

(b)

M1: Realises the requirement to cube the bracket and divide through by u and makes progress with the integration to obtain at least 3 terms of the required form e.g. 3 from ku^3, ku^2, ku, $k\ln u$

A1: Correct integration. This mark can be scored with the "2" still outside the integral or even if it has been omitted. But if the "2" has been combined with the integrand, the integration must be correct.

dM1: Completes the process by applying their "changed" limits and subtracts the right way round **Depends on the first method mark.**

A1: Cao (Allow equivalents for $\frac{104}{3}$ e.g. $\frac{208}{6}$)

Question T8_Q16

Question	Scheme	Marks	AOs
14(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2.1
	$1200\frac{dh}{dt} = 24 - 5h$*	A1*	1.1b
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24 - 5h} dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24 - 5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24 - 5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24 - 5h)$ oe	M1	3.1a
	$t = -240\ln(24 - 5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24 - 10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24 - 5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24 - 5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24 - 5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24 - 5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \to h = 4.8$ • As $e^{-\frac{t}{240}} > 0$, $h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	

(12 marks)
Notes

(a)

B1: Identifies the correct expression for $\frac{dV}{dt}$ according to the model

B1: Identifies the correct expression for $\frac{dV}{dh}$ according to the model

M1: Applies $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ or equivalent correct formula with their $\frac{dV}{dt}$ and $\frac{dV}{dh}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{dV}{dt} = 0.48 - 0.1h \Rightarrow \frac{dh}{dt} = \frac{0.48 - 0.1h}{24} \Rightarrow 1200\frac{dh}{dt} = 24 - 5h$ * scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{dt}{dh}$ correctly in terms of *h* **and** integrates to obtain $t = \alpha \ln(24 - 5h)(+c)$ or equivalent (condone missing brackets around the "24 – 5*h*") and + *c* not required for this mark.

A1: Correct equation in any form and + *c* not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes *t* = 0 and *h* = 2 to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain *h* in terms of *t*.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t + c = -240\ln(24 - 5h) \Rightarrow -\frac{t}{240} + d = \ln(24 - 5h) \Rightarrow Ae^{-\frac{t}{240}} = 24 - 5h$$

$$t = 0, h = 2 \Rightarrow A = 14 \Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = 4.8 - 2.8e^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $Ae^{\alpha t} = 24 - 5h$ (must have a constant "A")

A1: $Ae^{-\frac{t}{240}} = 24 - 5h$

ddM1: Uses *t* = 0, *h* = 2 in an expression of the form above to find *A*

A1: $h = 4.8 - 2.8e^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.
There must be no incorrect working or contradictory statements.
This is not a follow through mark and if their equation in (b) is used it must be correct.