Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 6: Exponentials and logarithms

Contents

Questions

Question T6_Q1

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

Question T6_Q2

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

Question T6_Q3

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

Question T6_Q4

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V\text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

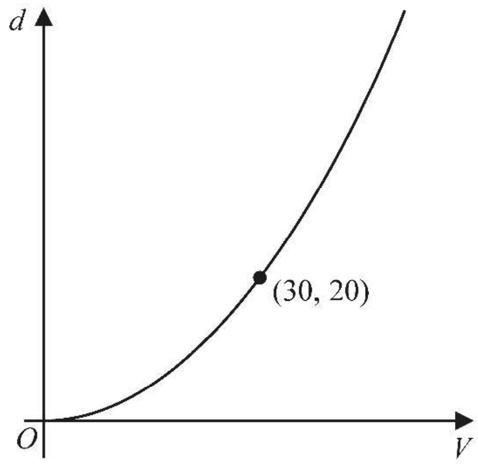

Figure 5

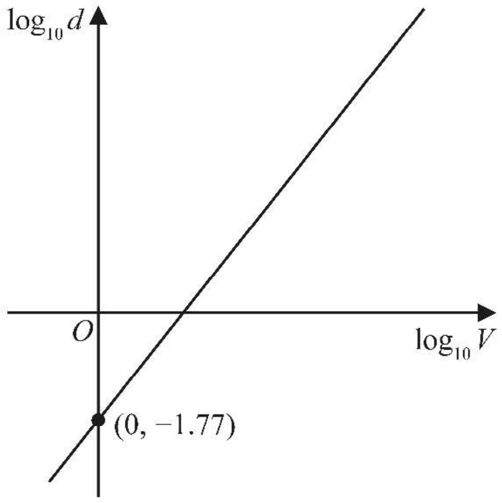

Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

Question T6_Q5

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

Question T6_Q6

8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

(2)

Question T6_Q7

3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

Question T6_Q8

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - B\mathrm{e}^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78°C

(b) Use this information to evaluate the model.

(2)

Question T6_Q9

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

Question T6_Q10

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

Question T6_Q11

10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

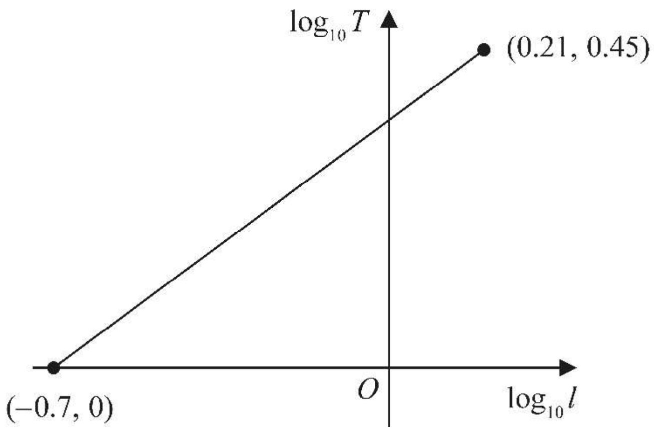

Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

Mark Scheme

Question T6_Q1

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5$, $50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or '≈ 24800'. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.

The statement must reference **the car**, its **cost/value**, and **"0" time**

Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.

The statement must reference **a yearly rate** and **an increase in value or multiplier**.

They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."

Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'

Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$

Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\frac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

Question T6_Q2

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20\,000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16\,000 \Rightarrow 16\,000 = 20\,000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20\,000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20\,000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18\,000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) Option 1

M1: **For** $V = Ae^{\pm kt}$ Do not allow if *k* is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20\,000$ into their exponential model

Candidates may start by simply writing $V = 20\,000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16\,000 = 20\,000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20\,000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20\,000e^{t \ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of *V* when *t* = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares *V* = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of *t* with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20\,000e^{-0.223t} \Rightarrow 2000 = 20\,000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to *k* then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000 r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when t = 10. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of r in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20\,000 \Rightarrow A = 18\,000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16000 = 2\,000 + 18000e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2000$

(b)

M1: Uses their model to find the value of V when t = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of k or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of k or the -0.251 greater (or less negative)'

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Question T6_Q3

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(a)

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make 'a' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in a are on the same side of the equation and the term in b is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$.
They may state that b cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)
As a minimum accept that $b > 1$ as a cannot be negative.
Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

Question T6_Q4

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10}(kV^n)$ $\Rightarrow$ $\log_{10} d = \log_{10} k + \log_{10} V^n$ $\Rightarrow$ $\log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ or $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}$ $20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = \ldots$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = \ldots$		
	$\{n = \text{awrt } 2.08 \Rightarrow\}$ $d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	(3)	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)
ADVICE: Ignore labelling (a), (b), (c) when marking this question Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part			

Notes for Question 9	
Note:	In their solution to (a) and/or (b) condone writing log in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their d to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of d
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their d to find a total stopping distance and gives a correct ft conclusion **or** finds their d and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8 \times$ their velocity
Note:	A thinking distance of awrt 13 and a value of d in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Question T6_Q5

Question	Scheme	Marks	AOs
2	$4^{3p-1}=5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \dfrac{210\log 5}{\log 4}+1 \Rightarrow p = ...$	dM1	2.1
	$p =$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.
Look for $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = \dfrac{210\log 5}{\log 4} \pm 1 \Rightarrow p = ...$ condoning slips.
You may see numerical versions E.g. $(3p-1)\times 0.60 = 210\times 0.7 \Rightarrow 1.8p - 0.6 = 147 \Rightarrow p = 82$
Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = 210\log\dfrac{5}{4} \pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

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There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210} \Rightarrow \dfrac{4^{3p}}{4} = 5^{210}$
but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

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Using base 4 or 5

E.g. $4^{3p-1}=5^{210} \Rightarrow (3p-1) = \log_4 5^{210}$

The M mark is not scored until $(3p-1) = 210\log_4 5$

Question T6_Q6

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

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You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

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Question T6_Q7

3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) $(x=)$ 1, 8	B1	1.1b
	(ii) 8 is not a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out. There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

(a) Alternative:
B1: Writes $x^2 - 9x + 8 = 0$ as $(4 - x)^2 - x - 8 = 0$ or equivalent
M1: Proceeds correctly to reach $\log(4-x)^2 = \log(x+8)$
A1: Obtains $2\log(4-x) = \log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)
B1: Writes down (x =) 1, 8
B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.
They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to log($4 - x$) or log of lhs or log(–4) or the 4 – 8. Some acceptable reasons are: log(–4) can't be found/worked out/is undefined, log(–4) gives math error, log(–4) = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with 4 – 8
Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation
There must be no contradictory statements.

Note that this is an independent mark but must have $x = 8$ (i.e. may have solved to get $x = -1, 8$ for first B mark)

Question T6_Q8

Question	Scheme	Marks	AOs
9(a)	$t=0,\ \theta=18 \Rightarrow 18=A-B$ **or** $t=10,\ \theta=44 \Rightarrow 44=A-Be^{-0.7}$	M1	3.1b
	$t=0,\ \theta=18 \Rightarrow 18=A-B$ **and** $t=10,\ \theta=44 \Rightarrow 44=A-Be^{-0.7}$ **and** $\Rightarrow A=...,B=...$	M1	3.1a
	At least one of: A = 69.6, B = 51.6 but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta=69.6-51.6e^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Notes:

(a)

M1: Makes the first key step in the solution of the problem. Substitutes t = 0 and θ = 18 **or** t = 10 and θ = 44 into the equation of the model to obtain an equation connecting A and B.
Note that $18 = A - Be^0$ scores M0 unless $18 = A - B$ is seen or implied later.
If they do not obtain an equation in A and B using the first conditions e.g. they have 18 = A – 1 then they can score this mark if they substitute A = 19 directly into $44 = A - Be^{-0.7}$ as an equation in A and B is implied.

M1: Substitutes t = 0 and θ = 18 **and** t = 10 and θ = 44 to obtain 2 equations connecting A and B **and** then proceeds to solves their equations in A and B simultaneously to obtain values for both constants. Do not be too concerned with the processing as long as values for A and B are obtained.

A1(M1 on EPEN): For A = awrt 70 **or** B = awrt 52

A1: For $\theta=69.6-51.6e^{-0.07t}$ **Must be a <u>fully correct equation as shown</u> but allow recovery if seen in (b).**
Note that some candidates evaluate e^0 as 0 and so obtain A = 18 and then write $44 = 18 - Be^{-0.7}$ and solve for B.
Such attempts can score M1M0A0A0 only.

(b)

B1ft: Identifies A as the boiling point/maximum temperature in the model. Follow through their A.

B1ft: Makes a valid conclusion (valid/not valid, good/not good etc.) that refers to the 78 and includes a reference to a significant/large difference

Alternative provided their A < 78

B1ft: $\theta=69.6-51.6e^{-0.07t}=78 \Rightarrow 51.6e^{-0.07t}=69.6-78=-8.4$

$\Rightarrow e^{-0.07t}=-\frac{7}{43}$ and $\ln\left(-\frac{7}{43}\right)$ and makes a reference to the fact that the equation cannot be solved or e.g. cannot take log of a negative number. You can condone numerical slips in the calculation.

B1ft: Model is not appropriate as 69.6(°C) is much lower than 78(°C)

Minimum for both marks: The model is not appropriate as "69.6"(°C) is much lower than 78(°C)

Note that these marks are not available if their equation is solvable. Note also that B0B1 is not possible.

Question T6_Q9

Question	Scheme	Marks	AOs
8(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = ...$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4\times"0.339"T = \ln 2 + "0.339"t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000e^{...t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $ae^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000e^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000 \times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates αe^{kt} to βe^{kt} and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

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Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000e^{0.139t}$, and then writes at $t = 8$ $\frac{dN}{dt}$ = awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

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Question T6_Q10

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\frac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly.
Can also be awarded for using $2=\log_3 9$. This may be implied by e.g.
$\log_3 ... = 2 \Rightarrow ... = 9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a correct equation in any form **and** solve for y.

A1: Correct **exact** value. Allow equivalent fractions.

Guidance on how to mark particular cases:

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

Question T6_Q11

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow *t* rather than *T* and *A* rather than *a*.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$

$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \text{ *}$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for *b* (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of *a*.

E.g. substitutes one of the given points and their value for *b* into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of *a*. Allow slips in rearranging their equation but must be correct log work to find *a*.

Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of *a*.

E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the equation not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation