Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 4: Sequences and series

Contents

Questions

Question T4_Q1

11. (a) Use binomial expansions to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$

(6)

A student substitutes $x = \dfrac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \dfrac{1}{2}$

(1)

(c) Substitute $x = \dfrac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

Question T4_Q2

4. (i) Show that $\sum_{r=1}^{16}\left(3 + 5r + 2^{r}\right) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \qquad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

Question T4_Q3

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

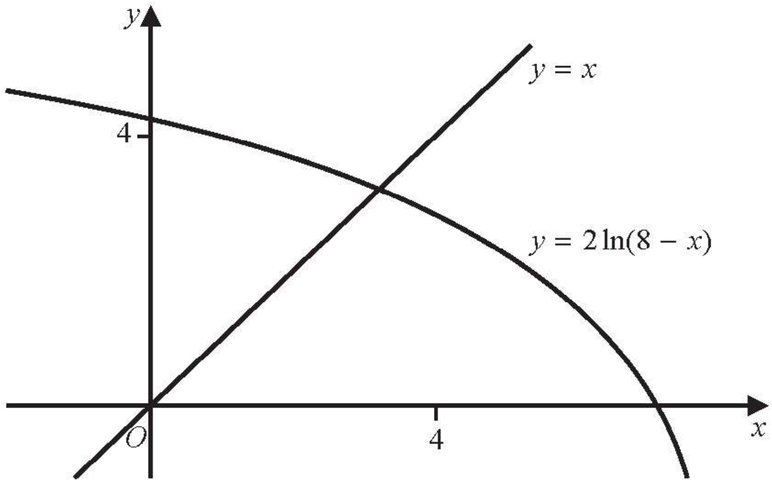

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

Question T4_Q4

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

Question T4_Q5

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

Question T4_Q6

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

Question T4_Q7

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

Question T4_Q8

13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

Question T4_Q9

4. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

Question T4_Q10

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

Question T4_Q11

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

Question T4_Q12

5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

Question T4_Q13

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

Question T4_Q14

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

Question T4_15

9. Show that

$$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

Mark Scheme

Question T4_Q1

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\dfrac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\dfrac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\dfrac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2......)\times\left(1+\dfrac{1}{2}x+\dfrac{3}{8}x^2...\right)$ $=1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\dfrac{5}{2}x-\dfrac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\dfrac{1}{4}$ Should not use $x=\dfrac{1}{2}$ as $\dfrac{1}{2}>\dfrac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\dfrac{1}{11}$ into $\sqrt{\dfrac{1+4x}{1-x}}\approx1+\dfrac{5}{2}x-\dfrac{5}{8}x^2$	M1	1.1b
	$\sqrt{\dfrac{3}{2}}=\dfrac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\dfrac{1183}{484}$ or $\dfrac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\dfrac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\dfrac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\dfrac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x|>\frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2}>\frac{1}{4}$ or stating that the expansion is only valid when $|x|<\frac{1}{4}$

Condone, for this mark a candidate who substitutes $x=\frac{1}{2}$ into the $4x$ **and** states it is not valid as $2>1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x=\frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}}\approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}}=\frac{1183}{968}$ oe $\sqrt{6}=2\times\frac{1183}{968}$

A1: $\sqrt{6}=\frac{1183}{484}$ or $\sqrt{6}=\frac{2904}{1183}$ $\quad\sqrt{6}=2\times\frac{1183}{968}=\frac{1183}{484}$ would imply all 3 marks

....

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5}\approx\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5}=1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}}=\sqrt{1+\frac{5x}{1-x}}=\left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}}\times\frac{\sqrt{1-x}}{\sqrt{1-x}}=\sqrt{(1+3x-4x^2)}\times(1-x)^{-1}=\left(1+(3x-4x^2)\right)^{\frac{1}{2}}\times(1-x)^{-1}$

Question T4_Q2

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, \ldots, : u_{n+1}=\dfrac{1}{u_n}, u_1=\dfrac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\dfrac{16}{2}(2(8)+15(5)) + \dfrac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=728+131\,070=131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\dfrac{16}{2}(2(5)+15(5)) + \dfrac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=48+680+131\,070=131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $= 10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1	3.1a
		M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{u_1=\dfrac{2}{3}\right\}, u_2=\dfrac{3}{2}, u_3=\dfrac{2}{3}, \ldots$ ***(can be implied by later working)***	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\dfrac{2}{3}\right)+50\left(\dfrac{3}{2}\right)$ or $50\left(\dfrac{2}{3}+\dfrac{3}{2}\right)$	M1	2.2a
	$=\dfrac{325}{3}$ $\left(\text{or } 108\dfrac{1}{3} \text{ or } 108.\dot{3} \text{ or } \dfrac{1300}{12} \text{ or } \dfrac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes for Question 4	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16}(3+5r+2^r)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$, $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3 + 5\sum_{r=1}^{16}r+\sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8,\ d=5,\ n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2,\ r=2,\ n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to 131 798
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$
(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is 131 798
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2}, \frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3}, \frac{3}{2}, \frac{2}{3}, \ldots$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2}, \frac{2}{3}, \ldots$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

Question T4_Q3

4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1\pm ax)^n=1\pm nax\pm\frac{n(n-1)}{2}a^2x^2+$

Condone sign slips and the "a" not being squared in term 3. Condone $a=\pm1$

Look for an attempt at the correct binomial coefficient for their n, being combined with the correct power of ax

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1+\frac{x}{8}+\frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2+..$

M1: For $4^{-\frac{1}{2}}+.....$ **M1:** As above but allow slips on the sign of x and the value of n **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x = -14$ with a suitable reason.

Eg. $x = -14$ as the expansion is only valid for $|x| < 4$ or equivalent.

Eg ' $x = -14$ as $|-14| > 4$"　　　or　　　' I cannot use $x = -14$ as $\left|\frac{-14}{4}\right| > 1$ '

Eg. ' $x = -14$ as is outside the range $|x| < 4$ '

Do not allow ' -14 is too big' or ' $x = -14, |x| < 4$ ' either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x = -\frac{1}{2}$ with a suitable reason.

Eg. $x = -\frac{1}{2}$ as it is 'the smallest/smaller value' or ' $x = -\frac{1}{2}$ as the value closest to zero' (that will give the more accurate approximation). The bracketed statement is not required.

Question T4_Q4

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^{1}$ 6th km is $6\times1.05\times1.05=6\times1.05^{2}$ 7th km is $6\times1.05\times1.05\times1.05=6\times1.05^{3}$ Hence the time for the r th km is $6\times1.05^{\,r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{\,r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the r th km and the index of 1.05
Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\dfrac{a\left(r^n-1\right)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.
Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum 6\times1.05^{n}$ or $24+\dfrac{6\left(1.05^{20}-1\right)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on a and n

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum

The value of r must be 1.05 oe such as 105% but you should allow a slip on the value of n used for their value of a (See below: We are going to allow the correct value of n or one less)

If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a = 6, \quad n = 17$ or 16 Eg. $\frac{6(1.05^{17} - 1)}{1.05 - 1} = (155.0)$ or $\frac{6(1.05^{16} - 1)}{1.05 - 1} = (141.9)$

Allow for $a = 6.3, n = 16$ or 15 Eg $\frac{6.3(1.05^{16} - 1)}{1.05 - 1} = (149.0)$ or $\frac{6.3(1.05^{15} - 1)}{1.05 - 1} = (135.9)$

Allow for $a = 6.615, \ n = 15$ or 14 Eg $\frac{6.615(1.05^{15} - 1)}{1.05 - 1} = (142.7)$ or $\frac{6.615(1.05^{14} - 1)}{1.05 - 1} = (129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24 + \frac{6.3(1.05^{16} - 1)}{1.05 - 1}$, $18 + \frac{6(1.05^{17} - 1)}{1.05 - 1}$, $30.3 + \frac{6.615(1.05^{15} - 1)}{1.05 - 1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

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Candidates that list values: Handy Table for Guidance

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6 + 6 + 6 + 6 + (6 \times 1.05) + ... + (6 \times 1.05^{16})$

M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

Km	Time per km	Total Time
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918
18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

Question T4_Q5

Question	Scheme	Marks	AOs
8 (i) **Way 1**	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \dots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) **Way 2**	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) **Way 3**	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) **Way 1**	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \dots\dots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\dots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) **Way 2**	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48}(\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \dots\dots + \log_5 50) - (\log_5 2 + \log_5 3 + \dots\dots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
		(6 marks)	

Notes for Question 8	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $\frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \ldots\ldots + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times \ldots \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}* • $\log_5\left(\frac{3}{2} \times \ldots \times \frac{49}{48} \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}*
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	<u>**Listing all 48 terms**</u> Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519\ldots + 0.1787\ldots + 0.1386\ldots + \ldots\ldots + 0.0125\ldots = 2$ {all terms in decimals}

Notes for Question 8	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2) - \log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 + ... + \log_5 49 + \log_5 50) - (\log_5 2 + ... + \log_5 48 + \log_5 49)$ • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 - \log_5 2) + (\log_5 4 - \log_5 3) + ... + (\log_5 50 - \log_5 49)$ • $\log_5 3 - \log_5 2, ..., \log_5 49 - \log_5 48, \log_5 50 - \log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$ $= \log_5(3 \times 4 \times .... \times 50) - \log_5(2 \times 3 \times .... \times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25 \times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

Question T4_Q6

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}}=1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of *x*. Do not accept ${}^{n}C_{r}$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^2+32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x,\ -8x^2, 32x^3$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or ≈ .

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}}=1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^2+32x^3$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\to\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

Question T4_Q7

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = ...$	M1	3.4
	= 62.8 (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = ...$ or $\frac{115}{"1.3265"}$	M1	3.4
	= 86.7 (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a + (n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\frac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2"d"$ or equivalent. This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r =$ awrt 1.33

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28 \times "r^4"$ or $\frac{115}{"r"}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28 \times "r^2"$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

Question T4_Q8

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2),\ a_2 = 2k,\ a_3 = k+1,\ a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2,\ a_{2/5} = -4,\ a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \frac{k(2+2)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2,\ a_2 = 2k,\ a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ and so $2 = \frac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26 \times (2 + -4 + -1) + 2 + -4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27 \times (2 + -4 + -1) - (-1)$

For candidates who find in terms of k award for $27 \times 2 + 27 \times (2k) + 26 \times (k+1)$ or $80k + 80$

If candidates proceed and substitute $k = -2$ into $80k + 80$ to get -80 then all 3 marks are scored.

A1: -80

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Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3} \times (2 + -4 + -1) = 26\frac{2}{3} \times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

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Question T4_Q9

Question	Scheme	Marks	AOs
4	${}^7C_4 a^3 (2x)^4$	M1	1.1b
	$\frac{7!}{4!3!}a^3 \times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.

The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3 (2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{4}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4}a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{4}a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3 (2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{3}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3}a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{3}a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2*x*" so allow e.g. $\frac{7!}{4!3!}a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4\times2$ or ${}^7C_4\times2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark.**

A1: $a = 3$ and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

Question T4_Q10

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + ... + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + ... + ar^n \Rightarrow S_n - rS_n = ...$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b

	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4 \times \dfrac{a(1-r^5)}{1-r}$ or $4 \times \dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1 - r$)	M1	3.1a
	$1 - r^{10} = 4(1 - r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = ...$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = ...$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of S.
The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n
Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by r and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.

Note: If terms are <u>listed</u> rather than <u>added</u> then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1 - r)$) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by a
Some candidates retain the "$1 - r$" and start multiplying out e.g. $(1 - r)(1 - r^{10})$ and this mark is still available as long as there is progress in cancelling the "a".

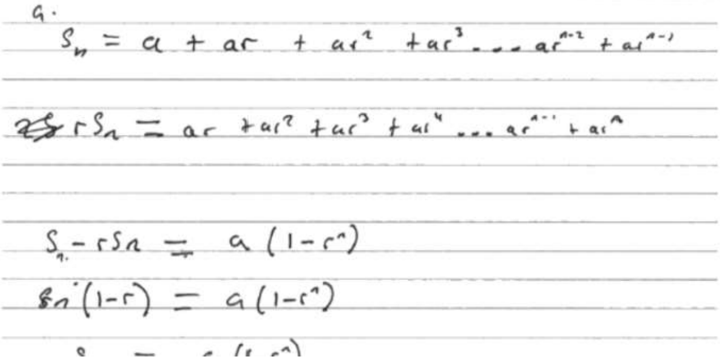
A1: Correct equation with the a and the $1 - r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = ...$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

> **(b) Note**: For candidates who use $S_5 = 4S_{10}$ expect to see:
> $4 \times \dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r} \Rightarrow 4(1-r^{10}) = (1-r^5)$ M1A0
> $4r^{10} - r^5 - 3 = 0 \Rightarrow (4r^5+3)(r^5-1) = 0 \Rightarrow r^5 = ...$ or $4(1-r^5)(1+r^5) = (1-r^5) \Rightarrow r^5 = ...$dM1A0

<u>Example for (a):</u>



This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised

Question T4_Q11

Question	Scheme	Marks	AOs
3(a)	$u_2 = k-12,\ u_3 = k - \dfrac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \dfrac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \dfrac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0*$	A1*	2.1
		(3)	
(b)	$k = 6, \left(\dfrac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k - \dfrac{24}{2}$ o.e. but could be awarded for $u_3 = k - \dfrac{24}{their\ "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \dfrac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of $k = 6$.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by $k =$
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least $k = 6$

A1: Chooses $k = 6$ and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\dfrac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

Question T4_Q12

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20000 \times 1.08^2 = (£)23328$*	B1*	1.1b
		(1)	
(b)	$20000 \times 1.08^{n-1} > 65000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
	Notes		

(a)
B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units
E.g. $£20000 \times 1.08^2$ or $£20000 \times 108\% \times 108\%$
This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.
Condone calculations seen as 8% of 20000 = 1600.
This is a show that question and the method must be seen.
It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.
Allow for example N or n for $n - 1$. So award for $20\,000 \times 1.08^{n-1} > 65\,000$,
$20\,000 \times 1.08^{n} = 65\,000$ or $20\,000 \times (108\%)^{n} \geqslant 65\,000$ amongst others.
Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula
The equation/inequality must contain an index of $n - 1, N, n$ etc.
Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000 \times 1.08^{n} = 65\,000 \Rightarrow n \log 1.08 = \log \frac{65000}{20000} \Rightarrow n = \ldots$

E.g. $2000 \times 1.8^{n} = 65\,000 \Rightarrow \log 2000 + n \log 1.8 = \log 65000 \Rightarrow n = \ldots$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list
E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16) \Rightarrow P = 20\,000 \times 1.08^{15}$ =awrt 63400 **or** $(n=17) \Rightarrow P = 20\,000 \times 1.08^{16}$ = awrt 68500

M1: $(n=16) \Rightarrow P = 20\,000 \times 1.08^{15}$ =awrt 63400 **and** $(n=17) \Rightarrow P = 20\,000 \times 1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.
You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n = 1.08 \times u_{n-1}$ seen at least 4 times and the sum attempted.
Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

Question T4_Q13

Question	Scheme	Marks	AOs
9(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B = \ldots$ or $C = \ldots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0 \Rightarrow 9 = 2A+B+4C$ $\Rightarrow 9 = 2A+1+8 \Rightarrow A = \ldots$	M1	2.1
	$A = 0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + \ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2} = 1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2} = \frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1+2x+\frac{-1(-1-1)}{2!}(2x)^2+\ldots$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x} = \frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$= \frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert < \frac{2}{5}$	B1	2.2a
		(7)	
		(11 marks)	

Notes

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x = \frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50 = -10A+25C$ which would be implied by $50 = -10A+25\times\text{"}2\text{"}$

Compare constant terms or substitute $x=0 \Rightarrow 9 = 2A+B+4C$ implied by $9 = 2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0 (b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+{*}x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \ldots$

M1: Correct attempt at the binomial expansion of $(1+{*}x)^{-2}$ up to the term in x^2

Look for $1+(-2){*}x+\frac{(-2)(-3)}{2}{*}x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2} + (-2)2^{-3} \times 5x + \frac{(-2)(-3)}{2} 2^{-4} \times (5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1){*}x+\frac{(-1)(-2)}{2}{*}x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5} < x < \frac{2}{5}$ or $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Question T4_Q14

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6 \Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
		(4 marks)	

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using a = 16, and n = 21 and the 24. The method may be implied by their working.

If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with a = 16, n = 500 and their numerical d from part (a)

If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

$$\text{(a) } d=\frac{24-16}{21}=\frac{8}{21} \quad \text{(b) } S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$$

This scores (a) M0A0 (b) M1A0

Question T4_Q15

Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
			(3 marks)
	Notes		

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be seen as part of them writing down the sequence but must be the **first** term.

M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$

A1*: Correct proof

Alternative 1:

B1: Deduces the correct value for the sum to infinity (starting at $n = 1$) or the common ratio

M1: Calculates the required value by subtracting the first term from their sum to infinity

A1*: Correct proof

Alternative 2:

B1: Deduces the correct value of the **first** term or the common ratio.

M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums

A1*: Correct proof

Alternative 3:

B1: Deduces the correct value of the **first** term

M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S"

A1*: Correct proof