Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 1: Proof

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Questions

Question T1_Q1

3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

Question T1_Q2

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

Question T1_Q3

16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

Question T1_Q4

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

Question T1_Q5

15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leqslant 4$

$$(n+1)^3 > 3^n$$

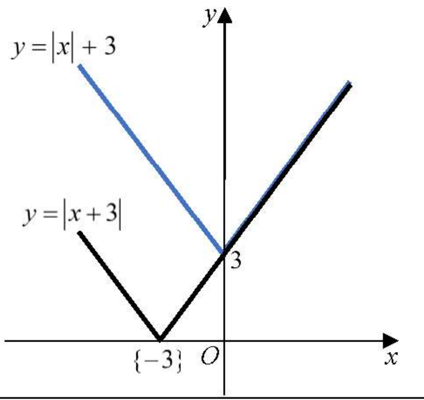
(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

Mark Scheme

Question T1_Q1

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3</td><td colspan="2">Statement: “If m and n are irrational numbers, where $m \neq n$,
then mn is also irrational.”</td><td></td><td></td></tr>
<tr><td rowspan="3">(a)</td><td colspan="2">E.g. $m = \sqrt{3},\ n = \sqrt{12}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">$\{mn =\}\ \left(\sqrt{3}\right)\left(\sqrt{12}\right) = 6$
$\Rightarrow$ statement untrue or 6 is not irrational or 6 is rational</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(b)(i),
(ii) Way 1</td><td rowspan="2">
</td><td>V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0,\ 3)$ or 3 stated or marked on the positive y-axis</td><td>B1</td><td>1.1b</td></tr>
<tr><td>Superimposes the graph of $y = |x+3|$ on top of the graph of $y = |x| + 3$</td><td>M1</td><td>3.1a</td></tr>
<tr><td rowspan="2"></td><td colspan="2">the graph of $y = |x| + 3$ is either the same or above the graph of
$y = |x+3|$ {for corresponding values of x}
or when $x \geq 0$, both graphs are equal (or the same)
when $x < 0$, the graph of $y = |x| + 3$ is above the graph of $y = |x+3|$</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="2">(b)(ii)
Way 2</td><td rowspan="2"><u>Reason 1</u>
When $x \geq 0,\ |x| + 3 = |x+3|$
<u>Reason 2</u>
When $x < 0,\ |x| + 3 > |x+3|$</td><td>Any one of Reason 1 or Reason 2</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Both Reason 1 and Reason 2</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="5" align="right">(5 marks)</td></tr>
</table>

<table>
<tr><th colspan="2">Notes for Question 3</th></tr>
<tr><td>(a)</td><td></td></tr>
<tr><td>M1:</td><td>States or uses any pair of different numbers that will disprove the statement.
E.g. $\sqrt{3},\ \sqrt{12}$; $\sqrt{2},\ \sqrt{8}$; $\sqrt{5},\ -\sqrt{5}$; $\frac{1}{\pi},\ 2\pi$; $3e,\ \frac{4}{5e}$;</td></tr>
<tr><td>A1:</td><td>Uses correct reasoning to disprove the given statement, with a correct conclusion</td></tr>
<tr><td>Note:</td><td>Writing $(3e)\left(\frac{4}{5e}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1</td></tr>
<tr><td>(b)(i)</td><td></td></tr>
<tr><td>B1:</td><td>See scheme</td></tr>
<tr><td>(b)(ii)</td><td></td></tr>
<tr><td>M1:</td><td>For constructing a method of comparing $|x| + 3$ with $|x+3|$. See scheme.</td></tr>
<tr><td>A1:</td><td>Explains fully why $|x| + 3 \geq |x+3|$. See scheme.</td></tr>
<tr><td>Note:</td><td>Do not allow either $x > 0,\ |x| + 3 \geq |x+3|$ or $x \geq 0,\ |x| + 3 \geq |x+3|$ as a valid reason</td></tr>
<tr><td>Note</td><td>$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1</td></tr>
<tr><td>Note:</td><td>Do not allow an incorrect statement such as $x \leq 0,\ |x| + 3 > |x+3|$ for A1</td></tr>
</table>

Notes for Question 3 Continued	
(b)(ii)	
Note:	Allow M1A1 for $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ and for $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert \geqslant$
Note:	Allow M1 for any of • x is positive, $\lvert x\rvert + 3 = \lvert x+3\rvert$ • x is negative, $\lvert x\rvert + 3 > \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • $x \ge 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • when $x \ge 0$, both graphs are equal • for positive values $\lvert x\rvert + 3$ and $\lvert x+3\rvert$ are the same Condone for M1 • $x \le 0,\ \lvert x\rvert + 3 > \lvert x+3\rvert$ • $x < 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$

(b)(ii) Way 3	• For $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • For $-3 < x < 0$, as $\lvert x\rvert + 3 > 3$ and $\{0 <\}\ \lvert x+3\rvert < 3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$	M1	3.1a
	• For $x \le -3$, as $\lvert x\rvert + 3 = -x + 3$ and $\lvert x+3\rvert = -x-3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$	A1	2.4

Question T1_Q2

Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up
(i)
M1: Awarded for setting up the proof for either the even or odd numbers.
A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.
dM1: Awarded for setting up the proof for both even and odd numbers
A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n=2m$, $n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1$, $n^2+2=(2m+1)^2+2=\ldots$ FYI $(4m^2+4m+3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2+m)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\dfrac{4m^2+2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\dfrac{n^2+2}{4} = \dfrac{4m^2+4m+3}{4} = m^2+m+\dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where
A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2} \times \sqrt{2k-1}$
dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question 10 (ii)	Scheme	Marks	AOs

(ii)

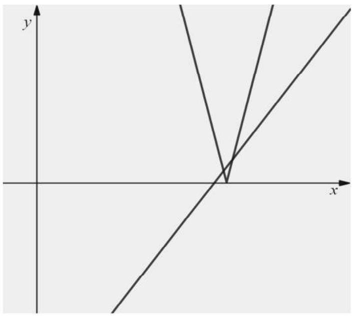
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\lvert 3x-28\rvert = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\lvert 3x-28\rvert = 8$ $x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\lvert 3x-28\rvert < x-9$ and points where $\lvert 3x-28\rvert > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x - 28 < x - 9$ and $-3x + 28 < x - 9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x - 28 \geqslant x - 9$ and $-3x + 28 \geqslant x - 9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Question T1_Q3

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p, q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*), or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question T1_Q4

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra. This must be made explicit e.g. reaches $3\times\left(3k^2+2k\right)+1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2+6k+1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2+6k+1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

Question T1_Q5

Question	Scheme	Marks	AOs
15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of n = 1, 2, 3 and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say n = 0, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for n =1, 2, 3 and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and **states** even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3-12p^2+6p+4=2(4p^3-6p^2+3p+2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if m^3+5 is odd then m is even"
- "this is contradiction, so proven."
- "So if m^3+5 is odd them m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a counter example to the statement "if m^3+5 is odd then m must be even" such as when $m=\sqrt[3]{2}$ then they can score special case mark B1